Spectrum of spontaneous emission into the mode of a cavity QED system

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We study the probe spectrum of light generated by spontaneous emission into the mode of a cavity QED system. The probe spectrum has a maximum on-resonance when the number of inverted atoms for an input drive is maximal. For a larger number of atoms N, the maximum splits and develops into a doublet, but its frequencies are different from those of the so-called vacuum Rabi splitting.

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Spontaneous emission in cavity Quantum Electrodynamics (QED) has generated considerable interest since the birth of the field [1]. The interaction between a single atom or N atoms and a single cavity mode is different from that in free space. The study of this interaction has led to ground-breaking experiments in nonlinear optics, squeezing, nonclassical correlations, and quantum information [2]. Spontaneous emission in cavity QED has been regarded as a dissipative process from which information is lost at a rate (γ) to modes other than the preferred cavity mode. Most work in cavity QED spontaneous emission has focused on the enhancement or suppression of the decay rate γ. An atom couples to the mode defined by the cavity mirrors through an allowed transition at a rate g. Photons escape the cavity due to imperfect mirror reflectivities at a rate 2κ. In the bad cavity limit ($\kappa >> \gamma, g$) the resonant spontaneous emission changes from its free space value as $\gamma \rightarrow \gamma(1+2C_1)$, with the single atom cooperativity $C_1 = g^2/(\kappa\gamma)$ [3]. The enhancement factor is related to the ratio of the atomic cross section to the cavity mode cross section multiplied by the average number of reflections inside the cavity. This effect broadens the spectrum, but causes no splitting [4]. There are many experimental demonstrations of enhanced and suppressed spontaneous emission in this regime (see for example the article by Hinds in Ref. [2]). If the reflectivity of the mirrors is high enough and the coupling between the atom and the cavity can become comparable to the two decays ($g \approx \kappa, \gamma$), spontaneous emission into the cavity is reversible. The total fraction of emission out of the cavity is $(1 + 2C_1')$; where $C_1' = C_1 2\kappa/(\gamma + 2\kappa)$. The factor $2\kappa/(\gamma + 2\kappa)$ is the fraction of photons emitted into the cavity mode, exiting the cavity via the mirror [5].

Spontaneous emission plays a dual role; it is a decohererence source, but it is also a way to extract information out of the system. An interrogation of the system through spontaneous emission is an unambiguous probe of the state of the atomic part of the atom-cavity system. This letter presents our investigations of the spectrum of light generated by a spontaneous emission process into the mode a driven optical cavity. The properties of the spontaneous emission channel in this system, together with the cavity output, are worth extensive study. Cavity QED has been identified as an environment to transfer information and entanglement between matter and light qubits [6]. The information has to exit the system through one of the two available channels as part of quantum interconnects and information protocols.

Our cavity QED system consists of a high finesse optical resonator where one or a few atoms interact with a single longitudinal and transverse mode of the cavity; the resulting coupling rate g depends on the dipole moment of the transition and the electric field that carries the energy of one photon. The combination of the rates in this system gives two dimensionless numbers, the first measures the effect of N atoms in the system through the cooperativity $C = C_1 N$ and the second measures the non-linearity of the system through the saturation photon number $n_0 = \gamma^2/8g^2$. The non-linearity is intrinsic in the system as a two-level atom in the excited state can not go further up the energy ladder, but only can come down through spontaneous or stimulated emission; in contrast, the cavity is a harmonic oscillator and its energy can increase without bound. The system is driven on-axis by a classical field ε/κ normalized normalized to photon flux units: $y = \varepsilon/(\kappa\sqrt{n_0})$. Its frequency ω_l is detuned from the atomic resonance ω_a and cavity resonance ω_c by an amount $\Omega = \omega_l - \omega_c$, with $\omega_a = \omega_c$. The atomic inversion σ^z is related to the expectation value of the intracavity field a and the collective atomic polarization σ_+ with their Hermitian conjugates through the equation of motion for their expectation values:

$$\frac{d < \sigma^z >}{\gamma dt} = < a\sigma^+ > + < a^\dagger\sigma^- > - (< \sigma^z > +1). \quad (1)$$

This equation shows that the atomic inversion is related to the correlation between the field in the cavity and the atomic polarization. In steady state the cross terms $< a\sigma^+ > + < a^\dagger\sigma^- >$ are proportional to the inversion. The probe spectrum (emission as function of driving field frequency) is related to the steady-state magnitude of the cross terms of Eq. 1 as a function of driving laser laser frequency.

The system can be accurately modelled, for weak excitation, as having either zero or one excitations of the coupled normal modes of the field and the atoms. If we assume fixed atomic positions, to first order in the excitation, $O(y^2)$, the equilibrium state is the pure state [3, 7]:

$$|\psi_{\rm ss}\rangle = |0, G\rangle + x\sqrt{n_0}|1, G\rangle - p\sqrt{n_0}|0, E\rangle + O(y^4). \quad (2)$$

Here $|n, G\rangle$ represents n photons with all (N) atoms in their ground state, $|n, E\rangle$ represents n photons with one atom in the excited state with the rest $(N-1)$ in their ground state. The small parameter is the expectation value of the intracavity field in the presence of atoms; $x\sqrt{n_0} = \langle a\rangle$. The induced atomic polarization of N atoms is $p = -2Cx$, which depends on the normalized input driving field y. We can pass now from Eq.1 to the semiclassical analysis of the weak field limit and use the steady state wave function to evaluate the inversion. To lowest order we have: $\langle a\sigma^+\rangle = \langle a\rangle\langle\sigma^+\rangle + O(y^4)$, which is equivalent to the decorrelation of the expectation values of the product of the field and the polarization [8], and we recover the Maxwell Bloch equations [9].

The spectrum of the transmitted light is given by the frequency dependent coefficients of the single excitation coefficients of the steady state (Eq. 2), those of order y. We can use the state equation of cavity QED to find the transmitted spectrum in both the field and the inversion.

The optical bistability literature [9] gives the relation between the expectation values of the field and polarization $< a >$, $< \sigma^+ >$ and measurable quantities as the normalized transmitted intensity, $X = |x|^2$, and the normalized incident intensity, $Y = |y|^2$. The transmitted and incident fields are related by the state equation $y = x(1+2C)$. The atoms respond to the external driving field by creating a polarization $p = - < \sigma^+ >$ that opposes that field and almost cancels it in the low intensity limit. In terms of the normalized fields (assuming equal phases as we are treating the resonant case), the total intensity in the absence of atoms (Y) in terms of the intensity in the presence of atoms (X) and the polarization is:

$$Y = |y|^2 = |x+p|^2 = |x|^2 + 2{\rm Re}(xp) + |p|^2 = X + F. \quad (3)$$

The total incoming energy Y goes out as transmission X or as fluorescence $F = 2{\rm Re}(xp) + |p|^2$. The fluorescence has two components, one is the magnitude of the polarization and the other a cross term between the intracavity field and the polarization $(2{\rm Re}(xp))$, similar to what we have in the equation of motion of the atomic inversion (Eq. 1). The intensity escaping though the cavity mode has a contribution from this term. It is rather difficult to separate from the drive and from stimulated emission. However, it contains non-trivial information about the atomic inversion, even in the case when $< \sigma_z >= -1 + O(x^2)$, at the low intensity limit $(x << 1)$.

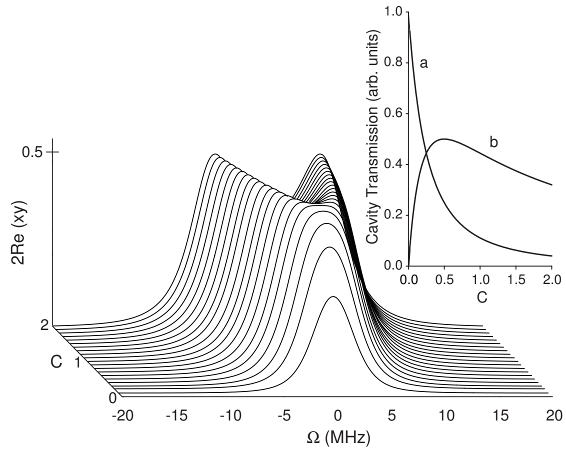

FIG. 1: Theoretical normalized probe spectrum of spontaneous emission into the cavity mode (equivalent to the atomic inversion spectrum) (2Re(xp)). for $\kappa/2\pi = 2.65$ MHz and $\gamma/2\pi = 6$ MHz. Inset resonant transmission for (a) the Vacuum Rabi spectrum and (b) atomic inversion spectra.

Figure 1 shows the theoretical calculation of the transmitted fluorescence spectrum. This is the cross term between the field and the polarization. It starts at zero for no atoms, then grows to a maximum on resonance that then develops a doublet. The peaks occur at $\Omega_{xp} = \pm\sqrt{g^2N - (\kappa^2 + (\gamma/2)^2)/2}$, a different value than those for the transmitted spectrum, which splits with peaks at: $\Omega_X = \pm\sqrt{g^2N\sqrt{1 + ((\gamma/g^2N)(\gamma/2+\kappa))} - (\gamma/2)^2}$ [10]. As the cooperativity grows $C >> 1$ the peaks of the fluorescence spectrum into the cavity mode approach those of the coupled atom-cavity system. The maximum on resonance $\Omega = 0$ occurs when a given drive y excites to the upper state an optimal number of atoms, before stimulated emission takes over moving the spectrum away from the center into an Autler-Townes-like doublet.

The inset in Fig.1 shows the normalized transmitted spectra on resonance for the transmitted intensity and the spontaneous emission. The transmitted intensity (a) starts at the peak of the transmission of the empty cavity and decreases monotonically with C, while the atomic inversion (b) starts at zero, grows and has a maximum for $C = 0.5$, and then decreases. The maximum coincides with the place where the spectrum splits into two peaks. The two peaks remain with a maximum value of 1/2 independent of C.

It is difficult to experimentally study the spontaneous emission in cavity QED. Work in the past has focussed on geometries that allow observation of the atoms from the side [11]. Another approach looks at the fluorescence into the mode of the cavity with the atoms driven by a laser that propagates perpendicular to the cavity axis [12, 13]. We follow Birnbaum *et al.* [14] to directly access a small part of the atomic inversion. We use the internal structure of the atoms to inform us when a transmit-

ted photon originates in a fluorescence event (see Fig. 2). Instead of utilizing Rb atoms in their stretched states ($m_F = F$ with $\Delta m = 1$) to form a closed two-level system when driven with circularly polarized light, we prepare the atoms into the $m_F = 0$ ground state and drive the optical transition with π polarization ($\Delta m = 0$). We can then look at the light emitted out of the cavity separating it into the two linear polarizations, one parallel to the drive and the other orthogonal to the drive. The presence of any light of orthogonal polarization signals that it comes originally from a spontaneous emission event of an atom that decays with $\Delta m \pm 1$.

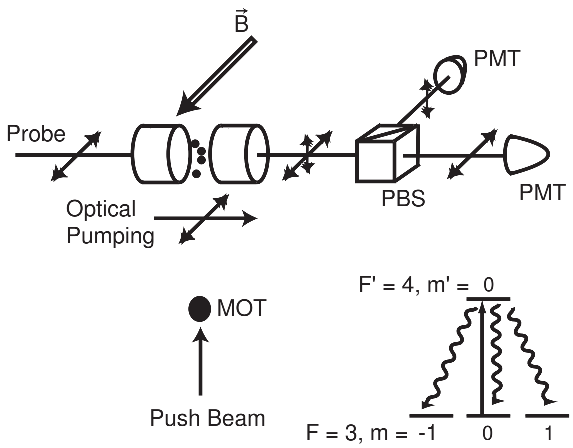

FIG. 2: Schematic of the experimental apparatus. A polarizer at the output separates the two orthogonal linear polarizations, one parallel to the driving field, the other perpendicular and coming from the decay through $\Delta m = \pm 1$ spontaneous emission.

The apparatus (see Fig. 2) consists of two main components: The source of atoms and the cavity. Two lasers provide the excitation radiation for the atomic source and for the cavity. A titanium sapphire laser (Ti:Sapph) provides most of the light needed for the experiment at 780 nm. The laser linewidth and long-term lock are controlled using a Pound-Drever-Hall technique on saturation spectroscopy of ^{85}Rb. A second laser repumps the atoms that fall out of the cycling transition in the trap.

A rubidium dispenser delivers Rb vapor to a magneto-optical trap (MOT) in a glass cell 20 cm below a cubic chamber that houses the cavity. The glass cell has a silane coating to decrease the sticking of Rb to the walls and maximize the capture efficiency of the MOT [15]. We use a six beam configuration with 1/e diameter of 20 mm (power) and 30 mW per beam. A pair of anti-Helmholtz coils generates a magnetic field gradient of 6 G/cm and three sets of independent coils zero the magnetic field at the trapping region.

The cavity defines a TEM_{00} mode with two 7 mm diameter mirrors with different transmission coefficients. The input transmission (15 ppm) is smaller than the output (250 ppm) to ensure that most of the signal escapes from the cavity on the detector side. The cavity finesse for this arrangement is $\mathcal{F} \approx 21\,000$ and its decay constant $\kappa/2\pi = 2.6$ MHz. The separation between the mirrors is 2 mm so the coupling coefficient between the mode and the dipole transmission of Rb is $g/2\pi = 2$ MHz. The mirrors are glued directly to flat piezo-electric-transducers (PZT) to control the length of the cavity.

Our experimental system is in the intermediate regime of cavity QED, where $g \approx (\kappa, \gamma/2)$. Its rates are $(g, \kappa, \gamma/2)/2\pi = (2, 2.6, 3.0)$ MHz. Since single atom cooperativity $C_1 = 0.25$ and the saturation photon number $n_0 = 1.1$ the system requires about one photon in steady state to become non-linear, but it starts to show the effects of spontaneous emission at a much lower intensity.

We lock the cavity with a Pound-Drever-Hall technique using a 820 nm laser. This laser is locked to the stabilized 780 nm laser using a transfer cavity. We separate the two wavelengths at the output of the physics cavity with a grating, and use appropriate interference filters to further ensure the separation of the two colors.

We launch the atoms from the MOT towards the cavity with a near-resonant probe beam from below. The repetition rate also sets the number of atoms delivered to the cavity. We take data by recording the transmitted light in the two orthogonal linear polarizations for a fixed excitation frequency Ω. There is a slight non-degeneracy of the two orthogonal modes of less than 0.5 MHz (less than the full width at half maximum of the transmission). The birefringence of the cavity is less than 1×10^{-4} on its axis.

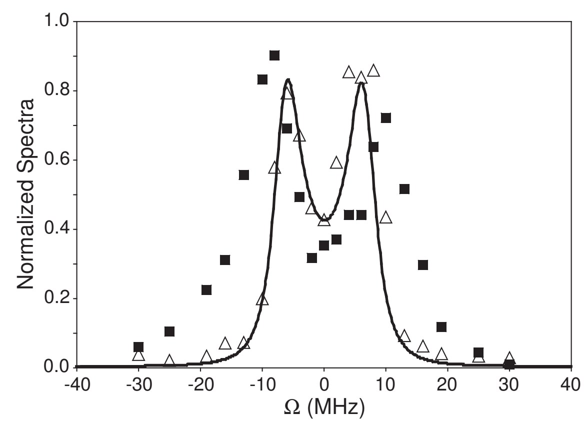

FIG. 3: Transmitted intensity spectrum: vacuum Rabi in filled squares; atomic inversion in empty triangles. The line shows the calculated spectrum for the spontaneous emission into the mode of the cavity with the height adjusted to match the normalized data.

The geometry that we use allows only π transitions ($\Delta m = 0$) and no Faraday rotation of the light since an external uniform magnetic field is aligned with the polarization direction of the incoming light. The observed light at the orthogonal polarization must come from spontaneous emission. This light is emitted into

the cavity mode so its detection is straightforward. The input drive Y is polarized horizontally to better than 1×10^{-5} and aligned to the magnetic field to better than ± 8 degrees.

As each launch of atoms (every 150 ms) traverses the cavity we record the transmission of both polarizations (parallel and orthogonal) in a digital storage scope. We then change the frequency of the driving laser and proceed to average over 200 launches of atoms. We extract from the raw data plots of the transmission spectrum for a given C. Figure 3 shows the transmitted spectrum for the two orthogonal polarizations, the one related to the intracavity field (X) in filled squares (parallel polarization) and the other the one related to the spontaneous emission in empty triangles (orthogonal polarization). The peaks of the spectrum for X are more separated than those of the spontaneous emission. The calculated spectrum for the spontaneous emission is normalized to match the data.

A detailed study of the susceptibility of this atomic system and its response with two orthogonal polarizations is beyond the scope of this letter and will be presented elsewhere [16]. We do not make any comments here about the predictions of the shape of the spectrum based on the simplifications of our model. Both polarizations have contributions from spontaneous emission, but only in the orthogonal polarization to the driving field the light coming from the decay of an excited atom is clearly identified.

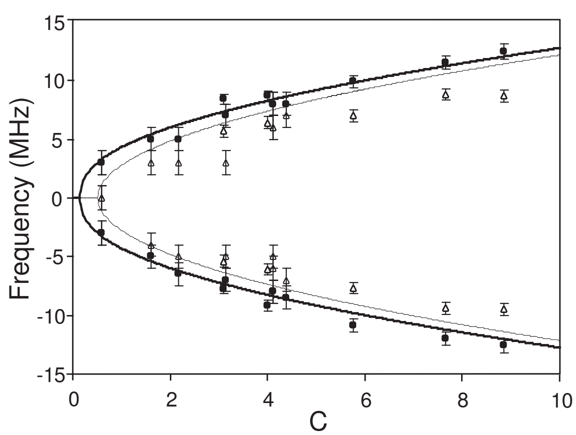

FIG. 4: Evolution of the position of the doublet splitting in the transmitted intensity. Filled squares show the intracavity intensity or vacuum Rabi splitting (parallel polarization); empty triangles the spontaneous emission spectrum (orthogonal polarization). The thick line is the prediction of Ω_X, while the thin line corresponds to Ω_{xp}.

Figure 4 presents a comparison of the position of the peaks in the measured spectra for both polarizations with the predictions of our simple theory as function of C which in our case varies because of the change in the number of atoms. The separation of the parallel polarization (cavity-atom doublet) starts earlier that the one in the orthogonal polarization (atomic inversion). For sufficiently high number of atoms, the positions of the two peaks coalesce into the same doublet in our simplified model. The overall horizontal scaling of this plot has been adjusted based on the relationship between C and Ω_X, and does not take into account any other broadening mechanisms. Fitting C to the size of the resonant transmitted light gives consistent results within 30% to those using Ω_X.

The study of the spectrum of spontaneous emission into the mode of a cavity QED system shows quantitatively different behavior from the vacuum Rabi spectrum. The labelling of the photons by polarization permits us to identify an emission out of the cavity generated by an excited atom spontaneous decay. This will allow measurements conditioned on the detection of a fluorescent photon in the future. The specific quantum dynamics of this photon with orthogonal polarization remain to be explored, in particular in the regime where we can no longer neglect higher order excitations.

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